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On $K \rightarrow \pi\pi$ Decays in Quenched and Unquenched Chiral Perturbation Theory

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We calculate the logarithmic corrections to the matrix elements for $K^+ \rightarrow \pi^+$ and $K \rightarrow \text{vacuum}$ (which are used on the lattice to determine $K \rightarrow \pi\pi$ amplitudes), in one-loop quenched and unquenched Chiral Perturbation Theory. We find that these corrections can be large. We also discuss, and present some results for, the direct determination of $K \rightarrow \pi\pi$ amplitudes. In particular, we address effects from choosing $m_s = m_d$ and vanishing external spatial momenta, finite volume and quenching. In the quenched octet case, we find *enhanced* finite-volume contributions which may make numerical estimates of this matrix element unreliable for *large* volumes.

1. Introduction

Chiral Perturbation Theory (ChPT) helps us understand several systematic errors which afflict lattice computations of $K \rightarrow \pi\pi$ decay amplitudes, and thus plays an important role in assessing the reliability of such computations. In particular, ChPT can be used to gain insight into the size of finite-volume and quenching effects, as well as the modifications induced by an unphysical choice of kinematics and/or the values of light quark masses. We consider two approaches to these amplitudes. The first is the direct computation of such amplitudes with $m_s = m_d$ and external mesons at rest [1]. Three key questions can be studied in ChPT: 1) how much do these unphysical choices affect the size of the chiral logarithms (with and without quenching)?; 2) are there quenched chiral logarithms [2]?; 3) are there *enhanced* finite-volume corrections? Here we mainly answer the last two questions, while a more detailed analysis will be given elsewhere [3]. Second, we calculate the $K \rightarrow \pi$ and $K \rightarrow 0$ (K to vacuum) matrix elements at one loop in ChPT, unquenched and quenched. The motivation comes from the possibility of performing an indirect determination of $K \rightarrow \pi\pi$ amplitudes through the computation of reduced matrix elements such as $K \rightarrow \pi$ and $K \rightarrow 0$, which is

simpler on the lattice [4].

2. The Unphysical $K^0 \rightarrow \pi^+ \pi^-$ amplitude

The Euclidean effective Lagrangian for $\Delta S = 1$ hadronic weak transitions can, at leading order in ChPT, be written as [4,5] (notation of [4]):

$$\mathcal{L}_{\Delta S=1} = -\alpha^{27} T_{kl}^{ij} (\Sigma \partial_\mu \Sigma^\dagger)^k_i (\Sigma \partial_\mu \Sigma^\dagger)^l_j \quad (1)$$

$$- \alpha_1^8 \text{tr}[\Lambda (\partial_\mu \Sigma) (\partial_\mu \Sigma^\dagger)] + \alpha_2^8 \frac{8v}{f^2} \text{tr}[\Lambda (\Sigma M + M^\dagger \Sigma^\dagger)],$$

where the first term transforms as $(27_L, 1_R)$ under $SU(3) \times SU(3)$ and the last two terms as $(8_L, 1_R)$.

The term with coupling α_2^8 is known as the “weak mass term,” and mediates the $K \rightarrow 0$ transition at tree level. Its odd-parity part, which in principle can also contribute to the octet $K \rightarrow \pi\pi$ amplitude, is proportional to $m_s - m_d$. For $m_s \neq m_d$ the weak mass term is a total derivative [4,6], and therefore does not contribute to any physical matrix element. Whether this term contributes to the octet $K \rightarrow \pi\pi$ matrix element for unphysical external momenta and $m_s = m_d$ is a more subtle question. What actually is computed on the lattice is the Euclidean correlation function $C(t_1, t_2) = \langle 0 | \pi^+(t_2) \pi^-(t_2) O_8(t_1) \bar{K}^0(0) | 0 \rangle$. Any contribution generated by the insertion of the weak mass term to the Euclidean correlation function at fixed times is proportional to $m_s - m_d$ in the limit $m_s \rightarrow m_d$ and therefore zero

*Presented by E. Pallante

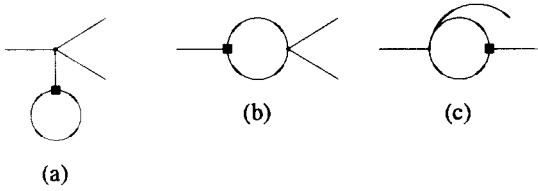


Figure 1. Some $K \rightarrow \pi\pi$ diagrams in ChPT. The box is a weak vertex, the dot a strong vertex.

at $m_s = m_d$; there are no subtleties with propagator poles in Euclidean space from tree-level tadpole diagrams [3]. This is also true for tadpole contributions as in diagram (a) of Fig. 1 with the insertion of an octet or a 27-plet weak operator. Such contributions are absent for $m_s = m_d$. We note that, choosing quark masses such that $m_K = 2m_\pi$, as proposed in [7], the contribution from α_2^8 vanishes for the same reason as for the physical $K \rightarrow \pi\pi$ amplitude, but that, in general, there are contributions from Fig. 1(a).

While the unphysical choice of kinematics and quark masses modifies the size of chiral logarithms and finite-volume corrections, quenching also causes new “quenched-artifact” contributions, due to the presence of the double pole in the singlet propagator. These are of two types: quenched chiral logarithms (Q χ L) and *enhanced* finite-volume corrections (discovered in quenched $\pi\pi$ scattering [8]). In principle, both artifacts occur in the quenched octet $K \rightarrow \pi\pi$ amplitude.

Only diagrams of type (b), (c) in Fig. 1 with the weak operator α_1^8 give rise to Q χ L in the unphysical $K^0 \rightarrow \pi^+\pi^-$ amplitude at $m_s = m_d$. However, we find that *no* Q χ L is present at one loop, due to a cancellation between contributions from type-(b) and type-(c) diagrams.

In finite volume, only the “rescattering diagram” of type (b) gives rise to power-like finite-volume corrections. It was shown in the case of $\pi\pi$ scattering [8] how, in the quenched approximation in a similar diagram, the presence of a double-pole singlet propagator gives rise to enhanced (infrared divergent!) finite-volume corrections. The same happens for the octet $K^0 \rightarrow \pi^+\pi^-$ amplitude.

We have calculated the chiral logs and power-like finite-volume corrections for $C(t_1, t_2)_{\text{octet}}$ to one loop. In the unquenched case we find

$$C(t_1, t_2) = \frac{8i\alpha_1^8 M^2 L^3}{f^3 (2M)^3} e^{-2M(t_2-t_1)-Mt_1} \left[1 - \mu(M) + \frac{7}{6} \frac{1}{f^2 L^3} (t_2 - t_1) - \frac{M^2}{(4\pi f)^2} \left(\frac{41.597}{ML} + \frac{62}{3} \frac{\pi^2}{(ML)^3} \right) \right], \quad (2)$$

where M is the degenerate meson mass, f is the pion decay constant in the chiral limit (normalized such that its value is 132 MeV at the physical pion mass), and $\mu(M) = (M^2/(16\pi^2 f^2)) \log(M^2/\Lambda^2)$ is the chiral logarithm. In the quenched case we obtain

$$C(t_1, t_2) = \frac{8i\alpha_1^8 M^2 L^3}{f^3 (2M)^3} e^{-2M(t_2-t_1)-Mt_1} \left[1 + \delta \left(-\frac{\pi^2}{M^2 L^3} (t_2 - t_1) - \frac{2\pi^2}{ML^3} (t_2 - t_1)^2 + \frac{3\pi^2}{4(ML)^3} + \frac{2.2284}{ML} - 0.41877ML \right) + 2\alpha\mu(M) + \frac{\alpha}{3} \frac{M^2}{(4\pi f_\pi)^2} \left(\frac{5\pi^2}{2(ML)^3} - \frac{14\pi^2}{M^2 L^3} (t_2 - t_1) + \frac{4\pi^2}{ML^3} (t_2 - t_1)^2 + \frac{31.198}{ML} + 0.83754ML \right) \right], \quad (3)$$

where $\delta = m_0^2/(24\pi^2 f^2)$ contains the singlet mass m_0 and α is another singlet parameter renormalizing its kinetic term [2].

We have ignored $O(p^4)$ contact terms, exponentially suppressed finite-volume effects, and contributions from excited states. The term linear in $t_2 - t_1$ can be related to finite-volume energy shifts of the two-particle internal states of type-(b) diagrams (at least in the unquenched case). The term linear in ML inside the square brackets of Eq. (3) is the enhanced finite-volume contribution, which is a quenched artifact, as is the term quadratic in $t_2 - t_1$.

3. $K^+ \rightarrow \pi^+$ and $K \rightarrow 0$ matrix elements

Ref. [4] proposed an indirect determination of the $K \rightarrow \pi\pi$ amplitudes with $\Delta I = 1/2$ and $3/2$

by computing on the lattice the reduced amplitudes $K^+ \rightarrow \pi^+$ and $K \rightarrow 0$. The ratio of the $\Delta I = 1/2$ and $3/2$ $K \rightarrow \pi\pi$ amplitudes can then be determined at tree level in ChPT through

$$\frac{[K^0 \rightarrow \pi^+\pi^-]_{\frac{1}{2}}}{[K^0 \rightarrow \pi^+\pi^-]_{\frac{3}{2}}} = \frac{[K^+ \rightarrow \pi^+]_{\frac{1}{2}} - b[K^0 \rightarrow 0]_{\frac{1}{2}}}{[K^+ \rightarrow \pi^+]_{\frac{3}{2}}}, \quad (4)$$

where $b = iM^2/f(m_K^2 - m_\pi^2)$, M is the degenerate mass used to compute $K^+ \rightarrow \pi^+$, and m_K, m_π are the nondegenerate masses used to compute $K \rightarrow 0$. The question arises how one-loop corrections modify Eq. (4). This problem was already addressed in [6] in the unquenched case, however, what is calculated there is the full pseudoscalar two-point function, and not the amplitude $K \rightarrow \pi$.

Here, we present the chiral logs for $K^+ \rightarrow \pi^+$ and $K \rightarrow 0$. For $K^+ \rightarrow \pi^+$, with degenerate masses, we find for $\Delta I = 1/2$, unquenched,

$$\frac{[K^+ \rightarrow \pi^+]}{4M^2/f^2} = \alpha_1^8 \left(1 - \frac{1}{3}\mu(M)\right) - \alpha_2^8 \left(1 + \frac{4}{3}\mu(M)\right) - \alpha^{27}(1 - 12\mu(M)), \quad (5)$$

while in the quenched case we obtain

$$\frac{[K^+ \rightarrow \pi^+]}{4M^2/f^2} = \alpha_1^8 \left(1 - 2\delta \log \frac{M^2}{\Lambda^2} + 4\alpha\mu(M)\right) - \alpha_2^8 \left(1 + \frac{4}{3}\alpha\mu(M)\right) - \alpha^{27}(1 - 6\mu(M)). \quad (6)$$

The $\Delta I = 3/2$ amplitudes are obtained from this by setting $\alpha_1^8 = \alpha_2^8 = 0$. Note that the contribution from ordinary chiral logarithms is substantially reduced by quenching. One should keep in mind that the values of the α 's are in principle different in the quenched and unquenched theories. The one-loop $K \rightarrow 0$ amplitude to leading order in $m_K^2 - m_\pi^2$ is (with M^2 some average of m_K^2 and m_π^2), unquenched,

$$\frac{[K \rightarrow 0]f}{4(m_K^2 - m_\pi^2)} = i\alpha_2^8 \left(1 - \frac{13}{3}\mu(M)\right) + i\alpha_1^8 \frac{10}{3}\mu(M),$$

while the quenched amplitude is

$$\frac{[K \rightarrow 0]f}{4(m_K^2 - m_\pi^2)} = i\alpha_2^8 \left(1 - \frac{4}{3}\alpha\mu(M)\right) + i\alpha_1^8 \left(2\delta \log \frac{M^2}{\Lambda^2} - 4\alpha\mu(M)\right).$$

Again, the chiral logarithms are potentially large, and reduced by quenching. One can now in principle extract unquenched

$$\frac{\alpha_1^8}{\alpha^{27}} = \frac{[K^+ \rightarrow \pi^+]_8(1 - 3\mu) - b[K \rightarrow 0](1 + \frac{8}{3}\mu)}{-[K^+ \rightarrow \pi^+]_{27}(1 + 12\mu)},$$

and quenched

$$\frac{\alpha_1^8}{\alpha^{27}} = \frac{[K^+ \rightarrow \pi^+]_8 - b[K \rightarrow 0](1 + \frac{8}{3}\alpha\mu)}{-[K^+ \rightarrow \pi^+]_{27}(1 + 6\mu)},$$

where $\mu \equiv \mu(M) = (M^2/(16\pi^2 f^2)) \log(M^2/\Lambda^2)$. It is clear that one-loop corrections are potentially large in the determination of weak-Lagrangian parameters from lattice computations. (For $M = 400$ MeV, $\Lambda = m_\rho$ and $f = 132$ MeV, $\mu(M) = -0.076$, and e.g. $1 + 6\mu = 0.54$.)

Obviously, in order to go beyond these “leading-log” estimates, it is necessary to consider also the contributions of $O(p^4)$ LECs to $K^+ \rightarrow \pi^+$ and $K \rightarrow 0$. Then, it is worth looking for ratios less sensitive to one-loop effects, if such exist (considering also other channels like $K \rightarrow \eta$ and/or varying momenta or masses), and also, whether (combinations of) the $O(p^4)$ LECs that appear in $K \rightarrow \pi\pi$ amplitudes can be extracted from amplitudes with less external legs.

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